HEATING OF CONDENSED PARTICLES IN A GAS FLOW

A. L. Suris

UDC 66.047

A solution is obtained for the heating of condensed particles in a gas flow, from which one derives an expression for the temperature reduction and rate of cooling of the gas.

When a dispersed material is heated by a gas flow, such as in a spraying dryer, or in cooling reaction products with a disperse liquid, one needs to determine the particle heating and the rate of gas cooling.

An equation for transient heat conduction [1] can be used to describe the heating of spherical particles in a gas flow on the assumption of spherical symmetry:

$$\frac{\partial T}{\partial \tau} = \frac{a}{r} \cdot \frac{\partial^2}{\partial r^2}(rT) \tag{1}$$

subject to the initial condition

$$T(r,\ 0) = T_0 \tag{2}$$

and the boundary condition

$$\left[\frac{\partial T}{\partial r} + w(T - T_g)\right]_{r=\rho} = 0, \tag{3}$$

where $w = \alpha/\lambda$.

The gas temperature appearing in the boundary condition is here not a constant but is dependent on the particle size and time. If the apparatus used for the heating is thermally insulated and if the particles are of identical size, then the heat balance for the two-phase flow gives the gas temperature in the form

$$T_g = A - B \int_0^\rho r^2 T(r,\ \tau)\, dr, \tag{4}$$

where

$$A = \frac{C_{gm}^{T_1} T_1}{C_{gm}^{T}} + \frac{G\Delta H_0}{G_g C_{gm}^{T}}; \quad B = \frac{G}{G_g} \cdot \frac{C_{gm}}{C_{gm}^{T}} \cdot \frac{3}{\rho^3}.$$

From (4), we put (3) in the form

$$\left\{\frac{\partial T}{\partial r} + w\left[T - A + B\int_0^\rho r^2 T(r,\ \tau)\, dr\right]\right\}_{r=\rho} = 0. \tag{5}$$

The solution to (1), (2), and (5) is obtained by separating the variables and is put as

$$T = y + 2(T_0 - y)\sum_{n=1}^{\infty} \frac{1}{r} C_n \exp\left(-a\frac{\mu_n^2}{\rho^2}\tau\right) \sin\frac{\mu_n r}{\rho}, \tag{6}$$

where

$$y = \frac{3A}{3 + B\rho^3}; \quad C_n = \frac{\rho(\sin\mu_n - \mu_n \cos\mu_n)}{\mu_n(\mu_n - \sin\mu_n \cos\mu_n)},$$

Moscow Chemical Machinery Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 23, No. 6, pp. 1050-1053, December, 1972. Original article submitted February 14, 1972.

TABLE 1. Values of μ_i as Functions of the Parameters

μ_1

q	p = −0,9	−0,6	−0,4	0,1	1,1	2,1
0,1	0,626	1,096	1,299	1,656	2,076	2,320
0,5	0,885	1,252	1,428	1,749	—	—
0,8	1,038	1,358	1,518	1,815	—	—
1,1	1,171	1,456	1,602	1,879	2,224	2,429
1,5	1,328	1,576	1,708	1,960	2,371	2,437
2,1	1 533	1,742	1,854	2,076	2,359	2,531
3,1	1,824	1,985	2,075	2,242	2,484	2,626
4,1	2,074	2,201	2,272	2,414	2,601	2,716
6,1	2,499	2,573	2,615	2,701	2,811	2,879
8,1	2,859	2,892	2,912	2,949	2,996	3,026
10,1	3,176	3,172	3,170	3,165	3,159	3,156
13,1	3,592	—	3,497	3,439	3,368	3,326
16,1	3,943	—	3,759	3,659	3,541	3,470
20,1	4,275	—	4,01	3,879	3,723	3,627

μ_2

p	q = 0,1	4,1	8,1	12,1	16,1	20,1
−0,9	4,516	4,521	4,530	4,547	4,588	4,718
0,1	4,735	4,783	4,851	4,946	5,076	5,244
1,1	4,933	5,006	5,095	5,205	5,335	5,484
2,1	5,105	5,186	5,280	5,388	5,507	5,635
4,1	5,367	5,447	5,533	5,624	5,719	5,817
7,1	5,616	5,682	5,749	5,818	5,888	5,958

μ_3

p	q = 0,1	4,1	8,1	12,1	16,1	20,1
−0,9	7,738	7,739	7,740	7,742	7,743	7,745
0,1	7,867	7,877	7,888	7,900	7,915	7,932
1,1	7,991	8,008	8,027	8,048	8,072	8,099
2,1	8,108	8,130	8,155	8,182	8,212	8,245
4,1	8,313	8,341	8,372	8,404	8,439	8,476
7,1	8,548	8,578	8,609	8,641	8,674	8,709

μ_4

q	p = −0,9	0,1	1,1	2,1	4,1	7,1
0,1	10,91	11,00	11,09	11,18	11,34	11,55
8 1	10,91	11,01	11,11	11,20	11,37	11,58
20,1	10,92	11,02	11,13	11,23	11,41	11,63

μ_5

q	p = −0,9	0,1	1,1	2,1	4,1	7,1
0,1	14,07	14,14	14,21	14,28	14,41	14,59
8,1	14,07	14,15	14,22	14,29	14,43	14,61
20,1	14,07	14,15	14,23	14,31	14,45	14,63

μ_6

q	p = −0,9	0,1	1,1	2,1	4,1	7,1
0,1	17,23	17,28	17,34	17,40	17,51	17,66
8,1	17,23	17,29	17,35	17,40	17,52	17,67
20,1	17,23	17,29	17,35	17,41	17,53	17,69

TABLE 1 (continued)

μ_7

q	P					
	−0,9	0,1	1,1	2,1	4,1	7,1
0,1	20,38	20,43	20,47	20,52	20,62	20,75
20,1	20,38	20,43	20,48	20,53	20,63	20,77

μ_8

q	P					
	−0,9	0,1	1,1	2,1	4,1	7,1
0,1	23,52	23,57	23,61	23,65	23,73	23,85
20,1	23,52	23,57	23,61	23,66	23,74	23,86

and μ_n are the positive roots of the equation

$$\operatorname{tg}\mu_n = \frac{(q-\mu_n^2)\mu_n}{P\mu_n^2+q}, \quad q = wB\rho^4;\ P = w\rho - 1. \tag{6}$$

The values of the first eight roots are given in the Tables for wide ranges in the parameters.

We substitute (6) into (4) to get

$$T_g = A - \frac{G}{G_g}\cdot\frac{C_{pm}}{C_{gm}^T}\left[y - \frac{6}{\rho}(T_0 - y)\sum_{n=1}^{\infty} A_n\right], \tag{7}$$

where

$$A_n = \frac{\rho(\sin\mu_n - \mu_n\cos\mu_n)^2}{\mu_n^3(\sin\mu_n\cos\mu_n - \mu_n)}\exp\left(-\frac{a}{\rho^2}\mu_n^2\tau\right).$$

We differentiate (7) with respect to τ to get cooling rate during the particle heating as

$$\frac{dT_g}{d\tau} = -\frac{G}{G_g}\cdot\frac{C_{pm}}{C_{gm}^T}\cdot\frac{6}{\rho^3}(T_0 - y)\,a\sum_{n=1}^{\infty}\mu_n^2 A_n.$$

NOTATION

T_g, T_i are the current and initial gas temperatures;
G, G_g are the mass flow rates of the condensed phase and gas;
r, ρ are the current radius and the radius of the particle surface;
τ is the time;
$C_{gm}^{T_i}$, C_{gm}^T are the mean heat capacities between standard temperature and T_i or T_g respectively;
C_{pm} is the mean heat capacity averaged over the whole particle volume;
α is the heat transfer coefficient for particle and gas flow;
T_0, T are the initial and instantaneous temperatures of the particles;
a, λ are the thermal diffusivity and thermal conductivity of the particle material;
ΔH_0 is the initial enthalpy of the particle material reckoned from the standard temperature.

LITERATURE CITED

1. A. V. Lykov, Theory of Thermal Conductivity [in Russian], Vysshaya Shkola, Moscow (1967).